

Lesson 1

HW p. 113 – 114 # 5 – 12, 31, 33, 41, 43, 45, 47, 49, 58, 63, 67

Power rule-Tangent line equation

<http://study.com/academy/lesson/power-rule-for-derivatives-examples-lesson-quiz.html>

Using the limit definition- we found derivatives now the short cut

Power rule $f(x) = ax^n$

$$f'(x) = nax^{n-1}$$

Sum and Difference Rules for Derivatives

$$dy/dx[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Constant Multiple Rule

$$dy/dx[c \cdot f(x)] = c \cdot f'(x)$$

Use the power rule to find the derivative- REWRITE when necessary

$$F(x) = x^3$$

$$F(x) = 5x^3$$

$$G(x) = \sqrt[3]{x}$$

$$F(x) = \frac{1}{x^2}$$

$$F(x) = \frac{2}{x}$$

$$F(x) = \frac{4x^2}{5}$$

$$G(x) = 2\sqrt{x}$$

$$f(x) = \frac{1}{2\sqrt[3]{x^2}}$$

$$f(x) = -\frac{3x}{2}$$

$$F(x) = x^3 - 4x + 5$$

$$G(x) = \frac{x^4}{2} + 3x^3 - 2x$$

$$F(x) = \frac{x^3 - 3x^2}{x}$$

AP Calculus AB

Graphing Calculator Worksheet

Finding derivatives on the Graphing Calculator!

MATH #8 OR 2ND CALC #6

In 1-3, Use a graphing calculator to find the value of the derivative of each of the following at the given point.

1. $f(x) = 3\ln x$ at the point $(1,0)$

2. $f(x) = \ln(\sqrt{1 + \sin^2 x})$ at the point $(\frac{\pi}{6}, \ln \frac{\sqrt{5}}{2})$

3. $f(x) = xe^{-x}$ at $(1, \frac{1}{e})$

In 4-6; Use your graphing Calculator to find the derivative and then write the equation of the tangent line at the given x value.

4. $y = 4\cos x - x$ when $x = 0$.

5. $y = \frac{1}{\sqrt{2x+2}} + 4$ when $x = 1$.

6. $y = 3^x$ when $x = 3$.

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Name _____

AP Calculus AB 3.3 Worksheet

Remember:

SHOW ALL WORK!

$$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \frac{d}{dx}(e^x) = e^x$$

$$(f \cdot g)' = f' \cdot g + g' \cdot f \quad \left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

Find the derivative of each:

1. $f(x) = \sin x + x^2$

2. $f(x) = e^x (\cos x)$

3. $f(x) = \frac{e^x + 1}{x^2 - 3}$

4. $G(x) = (\ln x)(\tan x)$

5. $f(x) = \frac{\cos x}{x} + \tan x$

6. $y = (\ln x)(e^x)$

7. $h(x) = (3x^2 + 2x + 1)\cos x$

8. $h(x) = (x^2 + e^x)\tan x$

9. $f(x) = (x^2 + e^x)(\cos x + \ln x)$

10. $f(x) = \frac{\sin x}{\ln x}$

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$$11. y = x^5 + 5x^4 - 10x - 7$$

$$12. y = \frac{1}{2x^2} + \frac{4}{\sqrt{x}}$$

$$13. y = \frac{2}{\sqrt[3]{x}} + \frac{4}{x^2}$$

$$14. f(x) = \frac{2x+2}{2x+3}$$

$$15. h(x) = \sqrt[3]{27x^2}$$

$$16. y = \frac{x^2 + 3x + 2}{x^2 - 1}$$

$$17. y = \frac{x^2 + 3x + 2}{x^2}$$

$$18. f(x) = 2\sqrt[3]{x}(\sqrt{x} + 3)$$

19. Find the slope of the curve $y = x^2 - 4x$ at the points where it crosses the x-axis.

20. A population of 5,000 bacteria is introduced into a culture and grows according to the equation $P(t) = 500\left(1 + \frac{4t}{50+t^2}\right)$, where t is hours. Find the rate at which the population is growing when $t = 2$.

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To live a creative life, we must lose our fear of being wrong.

~Anonymous

AP Calculus AB

Section 3.3 DAY 3

1. Solve for a and b in order for $g(x)$ to be both continuous and differentiable at $x = 0$.

$$g(x) = \begin{cases} ax + b & ; x > 0 \\ 1 - x + x^2 & ; x \leq 0 \end{cases}$$

2. Solve for a and b in order for $f(x)$ to be both continuous and differentiable at $x = 1$.

$$f(x) = \begin{cases} x^2 + 2 & ; x \leq 1 \\ a\left(x - \frac{1}{x}\right) + b & ; x > 1 \end{cases}$$

3. For $a - d$, find $f'(2)$ given the following information:

$$\begin{aligned} g(2) &= 3 & g'(2) &= -2 \\ h(2) &= -1 & h'(2) &= 4 \end{aligned}$$

a) $f(x) = 2g(x) + h(x)$

b) $f(x) = 4 - h(x)$

c) $f(x) = g(x)h(x)$

d) $f(x) = \frac{g(x)}{h(x)}$

7. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1, 5)$ is

A) $13x - y = 8$

B) $13x + y = 18$

C) $x - 13y = 64$

D) $x + 13y = 66$

E) $-2x + 3y = 13$

8. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{4}$ times the rate at which x is increasing. What is the value of k ?

A) 3

B) 4

C) 6

D) 8

E) 12

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~Anonymous

9. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

- A) $\frac{1}{4}$ B) 1 C) 4 D) $\frac{1}{\sqrt{2}}$ E) $\frac{1}{2\sqrt{2}}$

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

- A) -2 B) $\frac{1}{6}$ C) $\frac{1}{2}$ D) 2 E) 6

11. Which of the following is an equation of the tangent line to $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

- A) $y = 8x - 5$ B) $y = x + 7$ C) $y = x + .763$ D) $y = x - .122$ E) $y = x - 2.146$

12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

- A) $(\frac{1}{2}, \frac{1}{2})$ B) $(\frac{1}{2}, \frac{1}{8})$ C) $(1, -\frac{1}{4})$ D) $(1, \frac{1}{2})$ E) (2, 2)

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AP Calculus AB Review Sheet for Chapter 3 test

1. A spring is bobbing up and down so that its position at any time $t \geq 0$ is given by $s(t) = -4 \sin t$.

- a) What is the initial position of the spring?
- b) Which way is the particle moving to start? Justify your response.
- c) At $t = \frac{\pi}{4}$, is the spring moving up or down? Justify your response.
- d) Is the spring speeding up or slowing down at $t = \frac{\pi}{4}$? Justify your response.

2. If $y = \sec x$, find $\frac{d^2y}{dx^2}$.

3. If $x(t) = t^2 - 8t + 12$ is a position of a particle moving along the x axis at time t , then

- a) Find the average velocity for the first 3 seconds.
- b) Find the velocity at $t = 4$ seconds.
- c) When is the object stopped?
- d) When is the acceleration of the object 0?
- e) When does the object change direction?
- f) When does the object slow down?
- g) When is the object moving left?

4. $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} =$

- A) 0 B) 1 C) $\sin x$ D) $\cos x$ E) nonexistent

5. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

- A) $y = 2x + 1$ B) $y = x + 1$ C) $y = x$ D) $y = x - 1$ E) $y = 0$

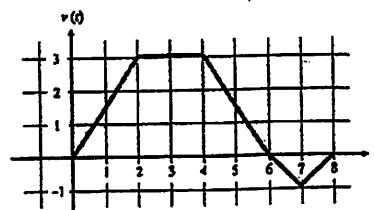
6. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

- A) $\sec x \csc x$ B) $\sec x - \csc x$ C) $\sec x + \csc x$ D) $\sec^2 x - \csc^2 x$ E) $\sec^2 x + \csc^2 x$

7. A bug begins to crawl up a vertical wire and its velocity at time t is given in the graph below.

Find:

- a) When does the particle change direction?
- b) When is the particle moving down?
- c) Is the particle speeding up or slowing down at $t = 6.5$?



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8. [Calculator] A particle moves along a line so that at time t , $0 \leq t \leq \pi$, its position is given by $s(t) = -4 \cos t - \frac{t^2}{2} + 10$.
What is the velocity of the particle when its acceleration is zero?

- A) -5.19 B) 0.74 C) 1.32 D) 2.55 E) 8.13

9. If $f(x) = \frac{x}{\tan x}$, then find $f'(2)$.

10. [Calculator]. Find the equation of the tangent line to the graph of $f(x) = (4 - \sin x)^2$ at $x = 0$.

11. Find $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec x - \sec(\frac{\pi}{4})}{x - \frac{\pi}{4}}$. This is a non-calculator question!! THINK!

12. Find y' if $y = 4e^x + 10 \ln x - \csc x + \sqrt[3]{x^2} + \frac{8}{x^2} + 4x - 100$

13. Given the data in the table find the following: (remember: $v(t) = s'(t)$)

t	1	2	3	4	5	6	7
S(t)	45	56	78	90	35	30	25

- a) $v(3)$ b) $v(4.5)$ c) Find the average velocity on the interval $[0,5]$.

14.

x	f(x)	g(x)	f'(x)	g'(x)
-1	0	-1	2	1

Given the functions f and g and their derivatives at $x = -1$ find $h'(-1)$.

- a) $h(x) = \frac{f(x)}{g(x)}$ b) $h(x) = f(x)g(x)$ c) $h(x) = 3f(x) - 4g(x)$

Also do in the book: page 149; 61



Name KELLY

AP Calculus AB 3.3 Worksheet

Remember:

SHOW ALL WORK!

$$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \frac{d}{dx}(e^x) = e^x$$

$$(f \cdot g)' = f' \cdot g + g' \cdot f \quad \left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

Find the derivative of each:

1. $f(x) = \sin x + x^2$
 $f'(x) = \cos x + 2x$

2. $f(x) = e^x (\cos x)$

$$f'(x) = (e^x)' \cos x + e^x (\cos x)'$$

$$f'(x) = e^x \cos x - e^x \sin x$$

3. $f(x) = \frac{e^x + 1}{x^2 - 3}$ $f'(x) = \frac{(x^2 - 3)e^x - (e^x + 1)(2x)}{(x^2 - 3)^2}$

4. $G(x) = (\ln x)(\tan x)$

$$G'(x) = \frac{1}{x} \cdot \tan x + \ln x \cdot \sec^2 x$$

$$= \frac{x^2 e^x - 3e^x - 2x e^x - 2x}{(x^2 - 3)^2}$$

5. $f(x) = \frac{\cos x}{x} + \tan x$

$$f'(x) = \frac{x(-\sin x) - \cos x \cdot 1}{x^2} + \sec^2 x$$

$$= \frac{-x \sin x - \cos x}{x^2} + \sec^2 x$$

6. $y = (\ln x)(e^x)$

$$y' = \frac{1}{x} e^x + \ln x \cdot e^x$$

7. $h(x) = (3x^2 + 2x + 1) \cos x$

$$h'(x) = (6x + 2) \cos x + (3x^2 + 2x + 1)(-\sin x)$$

$$(6x + 2) \cos x - (3x^2 + 2x + 1) \sin x$$

8. $h(x) = (x^2 + e^x) \tan x$

$$h'(x) = (2x + e^x) \tan x + (x^2 + e^x) \sec^2 x$$

9. $f(x) = (x^2 + e^x)(\cos x + \ln x)$

$$f'(x) = (2x + e^x)(\cos x + \ln x) + (x^2 + e^x)(-\sin x + 1/x)$$

10. $f(x) = \frac{\sin x}{\ln x}$

$$f'(x) = \frac{\ln x (\cos x) - \sin x}{(\ln x)^2}$$

$$f'(x) = \frac{x \ln x \cos x - \sin x}{x (\ln x)^2}$$

$$11. y = x^5 + 5x^4 - 10x - 7$$

$$y' = 5x^4 + 20x^3 - 10$$

$$12. y = \frac{1}{2x^2} + \frac{4}{\sqrt{x}} = \frac{1}{2}x^{-2} + 4x^{-1/2}$$

$$y' = -x^{-3} - 2x^{-3/2}$$

$$y' = -\frac{1}{x^3} - \frac{2}{x^{3/2}}$$

$$13. y = \frac{2}{\sqrt[3]{x}} + \frac{4}{x^2} = 2x^{-1/3} + 4x^{-2}$$

$$y' = -\frac{2}{3}x^{-4/3} - 8x^{-3}$$

$$\boxed{-\frac{2}{3x^{4/3}} - \frac{8}{x^3}}$$

$$15. h(x) = \sqrt[3]{27x^2} = 3x^{2/3}$$

$$h'(x) = 2x^{-1/3} = \frac{2}{x^{1/3}}$$

$$14. f(x) = \frac{2x+2}{2x+3}$$

$$f'(x) = \frac{(2x+3)(2) - (2x+2)(2)}{(2x+3)^2}$$

$$f'(x) = \frac{4x+6-4x-4}{(2x+3)^2} = \frac{2}{(2x+3)^2}$$

$$16. y = \frac{x^2+3x+2}{x^2-1}$$

$$y' = \frac{(x^2-1)(2x+3) - (x^2+3x+2)(2x)}{(x^2-1)^2}$$

$$y' = \frac{-3x^2-6x-3}{(x^2-1)^2}$$

$$17. y = \frac{x^2+3x+2}{x^2} = \frac{x^2}{x^2} + \frac{3x}{x^2} + \frac{2}{x^2}$$

$$= 1 + \frac{3}{x} + \frac{2}{x^2}$$

$$= 1 + 3x^{-1} + 2x^{-2}$$

$$y' = -3x^{-2} - 4x^{-3} = \boxed{-\frac{3}{x^2} - \frac{4}{x^3}}$$

$$18. f(x) = 2\sqrt[3]{x}(\sqrt{x}+3) = 2x^{1/3}(x^{1/2}+3) = 2x^{5/6} + 6x^{1/3}$$

$$f'(x) = \frac{10}{6}x^{-1/6} + 2x^{-2/3}$$

$$f'(x) = \frac{5}{3x^{1/6}} + \frac{2}{x^{2/3}}$$

19. Find the slope of the curve $y = x^2 - 4x$ at the points where it crosses the x-axis.

* $y = x^2 - 4x$ crosses the x-axis when $x=0$ and $x=4$.

$$y' = 2x - 4$$

$$y'|_{x=0} = 2(0) - 4 = \boxed{-4} \quad y'|_{x=4} = 2 \cdot 4 - 4 = 4$$

20. A population of 5,000 bacteria is introduced into a culture and grows according

to the equation $P(t) = 500\left(1 + \frac{4t}{50+t^2}\right)$, where t is hours. Find the rate at which the

population is growing when $t=2$.

$$P(t) = 500 + \frac{2000t}{50+t^2}$$

Need $P'(2)$

$$P'(t) = 0 + \frac{(50+t^2)(2000) - 2000t(2t)}{(50+t^2)^2}$$

$$P'(2) = \frac{(54)(2000) - 4000(4)}{(54)^2} = \frac{108000 - 16000}{2916} = \frac{92000}{2916} \approx 31.55$$

Bacteria per ho

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To live a creative life, we must lose our fear of being wrong.

~Anonymous
AP Calculus AB

Section 3.3 DAY 3

1. Solve for a and b in order for $g(x)$ to be both continuous and differentiable at $x = 0$.

$$g(x) = \begin{cases} ax+b & x > 0 \\ 1-x+x^2 & x \leq 0 \end{cases}$$

Dif. $\lim_{x \rightarrow 0^+} g'(x) = \lim_{x \rightarrow 0^+} a = a$
 $\lim_{x \rightarrow 0^-} g'(x) = \lim_{x \rightarrow 0^-} (-1+2x) = -1$
 $a = -1$

$$g(x) = \begin{cases} x^2+2 & x \leq 1 \\ a(x-\frac{1}{2})+b & x > 1 \end{cases}$$

Dif. $\lim_{x \rightarrow 1^+} f'(x) = \lim_{x \rightarrow 1^+} 2x = 2$
 $\lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^-} (2x-1) = 1$
 $2 = 1$
 $a = 1$

2. Solve for a and b in order for $f(x)$ to be both continuous and differentiable at $x = 1$.

$$f(x) = \begin{cases} x^2+2 & x \leq 1 \\ a(1-\frac{1}{x})+b & x > 1 \end{cases}$$

Cont. $f(1) = 1^2+2 = 3$
 $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (a(1-\frac{1}{x})+b) = a(1-1)+b = b$
 $3 = b$

For $a-d$, find $f'(2)$ given the following information:

a) $f(x) = 2g(x) + h(x)$
 $f'(x) = 2g'(x) + h'(x)$
 $f'(2) = 2g'(2) + h'(2) = 2(2) + 4 = 8$

b) $f(x) = 4 - h(x)$
 $f'(x) = 0 - h'(x)$
 $f'(2) = -h'(2) = -4$

c) $f(x) = g(x)h(x)$
 $f'(x) = g'(x)h(x) + h'(x)g(x)$
 $f'(2) = g'(2)h(2) + h'(2)g(2) = (-2)(-1) + (4)(3) = 2 + 12 = 14$

d) $f(x) = \frac{g(x)}{h(x)}$
 $f'(x) = \frac{g'(x)h(x) - h'(x)g(x)}{(h(x))^2}$
 $f'(2) = \frac{g'(2)h(2) - h'(2)g(2)}{(h(2))^2} = \frac{2(3) - (-1)(4)}{(-1)^2} = \frac{6+4}{1} = 10$

7. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1, 5)$ is

Quotient Rule

$$f'(x) = \frac{g'(x)h(x) - h'(x)g(x)}{(h(x))^2}$$

$$f'(2) = \frac{g'(2)h(2) - h'(2)g(2)}{(h(2))^2} = \frac{2(3) - (-1)(4)}{(-1)^2} = 10$$

8. When $x = 8$, the rate at which \sqrt{x} is increasing is $\frac{1}{4}$ times the rate at which x is increasing. What is the value of k ?

A) $13x - y = 8$
 $y' = \frac{2(3x-2) - 3(2x+3)}{(3x-2)^2}$
 $x = 8$
 $y' = \frac{2(24-2) - 3(16+9)}{(24-2)^2} = \frac{40 - 69}{22^2} = \frac{-29}{484}$
 $\frac{1}{4} = \frac{-29}{484} \cdot k$
 $k = \frac{484}{-29} \cdot \frac{1}{4} = -\frac{121}{29}$

B) $13x + y = 18$
 $y' = \frac{2(3x-2) - 3(2x+3)}{(3x-2)^2}$
 $x = 8$
 $y' = \frac{40 - 69}{22^2} = \frac{-29}{484}$
 $\frac{1}{4} = \frac{-29}{484} \cdot k$
 $k = \frac{484}{-29} \cdot \frac{1}{4} = -\frac{121}{29}$

C) $x - 13y = 64$
 $y' = \frac{2(3x-2) - 3(2x+3)}{(3x-2)^2}$
 $x = 8$
 $y' = \frac{40 - 69}{22^2} = \frac{-29}{484}$
 $\frac{1}{4} = \frac{-29}{484} \cdot k$
 $k = \frac{484}{-29} \cdot \frac{1}{4} = -\frac{121}{29}$

D) $x + 13y = 66$
 $y' = \frac{2(3x-2) - 3(2x+3)}{(3x-2)^2}$
 $x = 8$
 $y' = \frac{40 - 69}{22^2} = \frac{-29}{484}$
 $\frac{1}{4} = \frac{-29}{484} \cdot k$
 $k = \frac{484}{-29} \cdot \frac{1}{4} = -\frac{121}{29}$

E) $-2x + 3y = 13$
 $y' = \frac{2(3x-2) - 3(2x+3)}{(3x-2)^2}$
 $x = 8$
 $y' = \frac{40 - 69}{22^2} = \frac{-29}{484}$
 $\frac{1}{4} = \frac{-29}{484} \cdot k$
 $k = \frac{484}{-29} \cdot \frac{1}{4} = -\frac{121}{29}$

A) 3
 B) 4
 C) 6
 D) 8
 E) 12

(5)

To live a creative life, we must lose our fear of being wrong.

~Anonymous

$$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

twice rate at $x=1$

$$f'(c) = \frac{1}{2\sqrt{c}} = 2f'(1) = 2 \cdot \frac{1}{2\sqrt{1}} = 1$$

9. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x=c$ is twice its rate of change at $x=1$, then $c =$

A) $\frac{1}{4}$

B) 1

C) 4

D) $\frac{1}{\sqrt{2}}$

E) $\frac{1}{2\sqrt{2}}$

$$\frac{1}{2\sqrt{c}} = 1$$

$$1 = 2\sqrt{c}$$

$$(\sqrt{c})^2 = \left(\frac{1}{2}\right)^2$$

$$c = \frac{1}{4}$$

10. What is the instantaneous rate of change at $x=2$ of the function f given by $f(x) = \frac{x^2-2}{x-1}$?

A) -2

B) $\frac{1}{6}$

C) $\frac{1}{2}$

D) 2

E) 6

$$f'(x) = \frac{2x(x-1) - 1(x^2-2)}{(x-1)^2} = \frac{x^2 - 2x + 2}{(x-1)^2}$$

$$f'(2) = \frac{2^2 - 2(2) + 2}{(2-1)^2} = \frac{2}{1} = 2$$

11. Which of the following is an equation of the tangent line to $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

A) $y = 8x - 5$

B) $y = x + 7$

C) $y = x + .763$

D) $y = x - .122$

E) $y = x - 2.146$

$$f'(x) = 4x^3 + 4x = 1$$

$$4x^3 + 4x - 1 = 0 \leftarrow \text{use graphing calc. to solve}$$

12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

A) $(\frac{1}{2}, \frac{1}{2})$

B) $(\frac{1}{2}, \frac{1}{8})$

C) $(1, -\frac{1}{2})$

D) $(1, \frac{1}{2})$

E) (2, 2)

$$y - .1152 = 1(x - .2367)$$

$$x \approx .2367$$

$$f(.2367) \approx .1152$$

$$\frac{-4y}{-4} = \frac{-2x+3}{-4}$$

$$y = \frac{1}{2}x - \frac{3}{4}$$

$$m = \text{Slope} = \frac{1}{2}$$

$$y' = \frac{1}{2}(2)x' = x$$

$$x = \frac{1}{2}$$

$$y\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

$$\left(\frac{1}{2}, \frac{1}{8}\right)$$

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KEY

AP Calculus AB Review Sheet for Chapter 3 test

1. A spring is bobbing up and down so that its position at any time $t \geq 0$ is given by $s(t) = -4 \sin t$.

a) What is the initial position of the spring? $s(0) = 0$

b) Which way is the particle moving to start? Justify your response. $v(t) = -4 \cos t$

c) At $t = \frac{5\pi}{4}$, is the spring moving up or down? Justify your response.
 Down since $v(t) < 0$
 $0 - \pi/2 + \frac{3\pi}{2} - 2\pi$

up since $v(5\pi/4) > 0$

d) Is the spring speeding up or slowing down at $t = \frac{5\pi}{4}$? Justify your response. $a(t) = 4 \sin t$

$$v(5\pi/4) = +$$

$a(5\pi/4) = -$
 slowing down since the signs are different.

2. If $y = \sec x$, find $\frac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = \sec x \tan x$$

$$\frac{d^2y}{dx^2} = \sec x \tan^2 x + \sec^3 x$$

3. If $x(t) = t^2 - 8t + 12$ is a position of a particle moving along the x axis at time t , then

$$v(t) = 2t - 8$$

$$a(t) = 2$$

a) Find the average velocity for the first 3 seconds. -5 m/s

b) Find the velocity at $t = 4$ seconds. 0 m/s

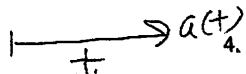
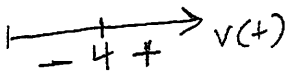
c) When is the object stopped? $t = 4 \text{ sec}$

d) When is the acceleration of the object 0? Never

e) When does the object change direction? $t = 4$

f) When does the object slow down? $0 < t < 4$

g) When is the object moving left? $0 < t < 4$
 since $v(t) < 0$
 $v(t) \neq a(t)$
 Diff sign!



4. $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} =$ asking for the deriv of $\sin x$ which is $\cos x$

A) 0

B) 1

C) $\sin x$

D) $\cos x$

E) nonexistent

5. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

$$y' = 1 - \sin x \quad m_T = 1 - \sin(0)$$

A) $y = 2x + 1$

B) $y = x + 1$

C) $y = x$

D) $y = x - 1$

E) $y = 0$

$$y - 1 = 1(x - 0) \\ y = x + 1$$

6. If $y = \tan x - \cot x$, then $\frac{dy}{dx} = \sec^2 x + \csc^2 x$

A) $\sec x \csc x$

B) $\sec x - \csc x$

C) $\sec x + \csc x$

D) $\sec^2 x - \csc^2 x$

E) $\sec^2 x + \csc^2 x$

7. A bug begins to crawl up a vertical wire and its velocity at time t is given in the graph below.

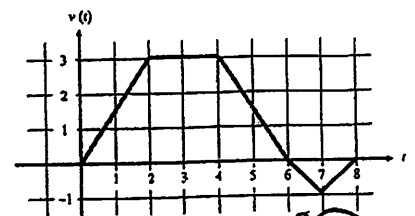
Find:

a) When does the particle change direction? $t = 6$

b) When is the particle moving down? $6 < t < 8$

c) Is the particle speeding up or slowing down at $t = 6.5$?

speeding up $v(t) \neq a(t)$ have the same sign!



7

★

$v(t) = 4\sin t - t$
 $a(t) = 4\cos t - 1$

8. [Calculator] A particle moves along a line so that at time t , $0 \leq t \leq \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

- A) -5.19 B) 0.74 C) 1.32 **D) 2.55** E) 8.13

$4\cos t - 1 = 0$
USE CALC
 $t \approx 1.318$

$v(1.318) = 2.554$

9. If $f(x) = \frac{x}{\tan x}$, then find $f'(2)$.

CALC!

$\frac{\tan x(1) - x \cdot \sec^2 x}{\tan^2 x} = \frac{\tan x - x \sec^2 x}{\tan^2 x} \approx -2.876$

10. [Calculator]. Find the equation of the tangent line to the graph of $f(x) = (4 - \sin x)^2$ at $x = 0$.

USE CALC TO FIND $f'(0)$! MATH #8

-8

11. Find $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec x - \sec(\frac{\pi}{4})}{x - \frac{\pi}{4}}$. This is a non-calculator question!! THINK!

They want the derivative of $y = \sec x$
 at $x = \pi/4$!
 $y' = \sec x \tan x$
 $y'(\pi/4) = \sec \pi/4 \tan \pi/4 = 1$

12. Find y' if $y = 4e^x + 10\ln x - \csc x + \sqrt[3]{x^2} + \frac{8}{x^2} + 4x - 100$

$y' = 4e^x + \frac{10}{x} + \csc x \cot x + \frac{2}{3}x^{-1/3} - 16x^{-3} + 4$

13. Given the data in the table find the following: (remember: $v(t) = s'(t)$)

t	1	2	3	4	5	6	7
S(t)	45	56	78	90	35	30	25

- a) $v(3) = s'(3) \approx \frac{90-56}{4-2} = 17$
 b) $v(4.5) = s'(4.5) \approx \frac{35-90}{5-4} = -55$
 c) Find the average velocity on the interval $[0,5]$. $\frac{s(5)-s(0)}{5-0} = \frac{35-45}{5} = -2$

14.

x	f(x)	g(x)	f'(x)	g'(x)
-1	0	-1	2	1

Given the functions f and g and their derivatives at $x = -1$ find $h'(-1)$.

$h'(-1) = 3f'(-1) - 4g'(-1) = 2$

- a) $h(x) = \frac{f(x)}{g(x)}$
 b) $h(x) = f(x)g(x)$
 c) $h(x) = 3f(x) - 4g(x)$
- $b) h'(-1) = f'(-1)g(-1) + f(-1)g'(-1) = (2)(-1) + 0(1) = -2$

Also do in the book: page 149; 61

$a) h'(-1) = \frac{g(-1) \cdot f'(-1) - f(-1) \cdot g'(-1)}{g^2(-1)} = \frac{(-1)(2) - (0)}{(-1)^2} = -2$

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