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Chapter 6: Integration Test Review

Problems from textbook: Page 320; Review exercises; #16-26 even, 30, 31
Page 319; QUICK QUIZ for AP at the top of the page #1-4
***Know how to approximate area under the curve or integrals using LRAM, MRAM, RRAM, Trapezoidal rule with functions and tables!**

1. Approximate $\int_0^4 (4-x^2)dx$ using LRAM, RRAM, and MRAM with n =8 (8 subintervals). Are each of these estimates under or over estimates? Then calculate the exact value of the integral using FTC.

LRAM: $0.5 \times 4 = 2$
$0.5 \times 3.75 = 1.875$
$0.5 \times 3 = 1.5$
$0.5 \times 1.75 = 0.875$
$0.5 \times 0 = 0$
$0.5 \times 2.25 = -1.125$
$0.5 \times -5 = -2.5$
$0.5 \times -8.25 = -4.125$
-1.5
overestimate

RRAM
$0.5 \times 3.75 = 1.875$
$0.5 \times 3 = 1.5$
$0.5 \times 1.75 = 0.875$
$0.5 \times 0 = 0$
$0.5 \times -2.25 = -1.125$
$0.5 \times -5 = -2.5$
$0.5 \times -8.25 = -4.125$
$0.5 \times -12 = -6$
-9.5
underestimate

MRAM
$0.5 \times 3.9375 = 1.96875$
$0.5 \times 3.4375 = 1.71875$
$0.5 \times 2.4375 = 1.21875$
$0.5 \times .9375 = 0.46875$
$0.5 \times -1.063 = -.5315$
$0.5 \times -3.563 = -1.7815$
$0.5 \times -6.563 = -3.2815$
$0.5 \times -10.06 = -5.03$
-5.2195

$$\int_0^4 (4-x^2)dx = -5.333$$

2. Use Trapezoidal rule to approximate $\int_0^{\pi} \sin x dx$ with n = 4 subintervals. Is this an under or overestimate? $0, \frac{\pi}{4}, \frac{\pi}{2}, \pi$

underestimate

$\frac{1}{2}(\frac{\pi}{4})(0.70711) = 0.27768$
$\frac{1}{2}(\frac{\pi}{4})(0.70711+1) = 0.67038$
$\frac{1}{2}(\frac{\pi}{4})(1+0.70711) = 0.67038$
$\frac{1}{2}(\frac{\pi}{4})(0.70711) = 0.27768$

1.89612

Underestimate because the graph is concave down on $(0, \pi)$

3. t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket *A* has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket *A* over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Using correct units, explain the meaning of $\frac{1}{80}\int_0^{80} v(t)dt$.

a) $\frac{49-5}{80-0} = \frac{44}{80} = \frac{11}{20} = 0.55\ ft/s^2$

b) $\int_{10}^{70} v(t)\,dt$ is the distance in feet that is traveled by Rocket A from 10 to 70 seconds.

MRAM: $20(22+35+44) = 2020$ ft

c) $\frac{1}{80}\int_0^{80} v(t)dt$ is the average distance in feet travelled by Rocket A per second.

4. The function f is given by $f(x) = \int_0^{3\sin x} \sqrt{4+t^2}\,dt$.

(a) Find $f'(x)$.

(b) Write an equation of the line tangent to the graph of f(x) at $x = \pi$.

(4.0) 4

a) $\frac{dy}{dx}\int_0^{3\sin x}\sqrt{4+t^2} = \sqrt{4+(3\sin x)^2}\cdot 3\cos x = 3\cos x\sqrt{4+9\sin^2 x}$

b) $f(\pi) = \int_0^{3\sin\pi}\sqrt{4+x^2} = \int_0^0 = 0$

$f'(\pi) = 3\cos\pi\sqrt{4+9\sin^2\pi} = 3(-1)(2) = -6$

$(\pi, 0)$

$y = -6(x-\pi)$

$y = -6x + 6\pi$

$g(x) = \int f(x)$
$g'(x) = f(x)$
$g''(x) = f'(x)$

5.

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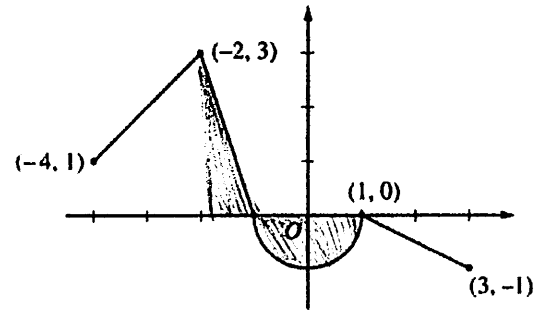
Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

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a) $g(2) = \frac{1}{2}(1)(-\frac{1}{2}) = \frac{-1}{4}$

$g(-2) = -\int_{-2}^{1} f(t) = -\left[\frac{1}{2}(1)(3) - \frac{1}{2}\pi(1)^2\right] = -\left(\frac{3}{2} - \frac{\pi}{2}\right) = -\frac{3}{2} + \frac{\pi}{2}$

b) $g'(-3) = f(-3) = 2$

$g''(-3) = f'(-3) = 1$

c) horizontal tangent where $g'(x) = f(x) = 0$

$x = -1$ relative maximum because g' changes from $+$ to $-$

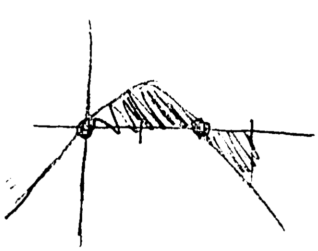
$x = 1$ neither because g' does not change signs

d) $x = -2, 0, 1$ because $g''(x) = f'(x)$ changes signs.

$\frac{2x^2}{2}$

$x(2-x)$

6. Find the total area between the curve $f(x) = 2x - x^2$ and the x-axis from x= 0 to 3.



$\int_0^2 (2x - x^2) + \left|\int_2^3 (2x - x^2)\right|$

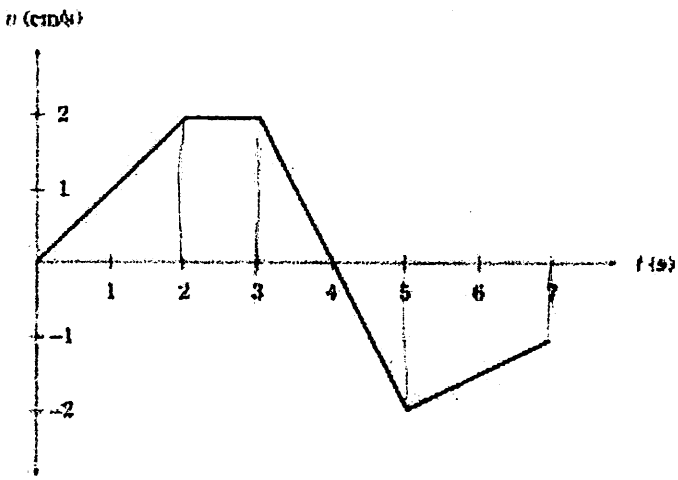
$\int_0^2 (2x - x^2) = x^2 - \frac{x^3}{3}\Big|_0^2 \; (2^2 - \frac{2^3}{3}) - 0 = 4 - \frac{8}{3} = \frac{4}{3}$

$\int_2^3 (2x - x^2) = x^2 - \frac{x^3}{3}\Big|_2^3 \; (3^2 - \frac{3^3}{3}) - (2^2 - \frac{2^3}{3}) = 0 - \frac{4}{3} = -\frac{4}{3}$

$\left|-\frac{4}{3}\right| = \frac{4}{3}$

area: $\frac{4}{3} + \frac{4}{3} = \frac{8}{3}$

7.



A particle moves along the x-axis so that its velocity at time t seconds, for $0 \le t \le 7$, is given by the function v(t) in cm/s shown above. At time t = 0 the particle is at x = -4 cm.

a) **What is the total distance traveled by the particle for $0 \le t \le 7$? Set up an integral expression to represent the total distance and then find the answer.**

$\int_0^7 |v(t)| = \int_0^4 v(t) + \left|\int_4^7 v(t)\right|$

$= \frac{1}{2}(2)(2)+2 + \frac{1}{2}(1)(2) + \frac{1}{2}(2) + \frac{1}{2}(2)(2+1) = 5+1+3 = 9$ cm

b) **What is the displacement of the particle on the interval $0 \le t \le 7$? Set up an integral expression to represent the displacement and then find the answer.**

$\int_0^7 v(t) = \frac{1}{2}(1+4)(2) + \frac{1}{2}(1)(-2) + \frac{1}{2}(2)(-2-1)$

$= 5 - 1 - 3 = 1$ cm

c) **What is the position of the particle at t = 7? Write an integral expression to represent the position and then find the answer.**

$x(7) = -4 + \int_0^7 v(t)$

$= -4 + 1 = -3$ cm

d) **When is the particle at rest? Justify your answer.**

The particle is at rest at t = 4 seconds because velocity is 0.

e) **When is the particle the farthest to the right? Justify your answer.**

$x(0) = -4$ cm

* $x(4) = \int_0^4 v(t) = \frac{1}{2}(1+4)(2) = 5$ cm

$x(7) = 1$ cm