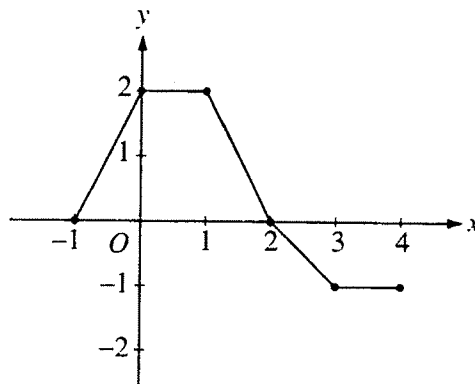


55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

1. What is the  $x$ -coordinate of the point of inflection on the graph of  $y = \frac{1}{3}x^3 + 5x^2 + 24$ ?

(A) 5                      (B) 0                      (C)  $-\frac{10}{3}$                       (D) -5                      (E) -10



2. The graph of a piecewise-linear function  $f$ , for  $-1 \leq x \leq 4$ , is shown above. What is the value of  $\int_{-1}^4 f(x) dx$ ?

(A) 1                      (B) 2.5                      (C) 4                      (D) 5.5                      (E) 8

3.  $\int_1^2 \frac{1}{x^2} dx =$

(A)  $-\frac{1}{2}$                       (B)  $\frac{7}{24}$                       (C)  $\frac{1}{2}$                       (D) 1                      (E)  $2 \ln 2$

4. If  $f$  is continuous for  $a \leq x \leq b$  and differentiable for  $a < x < b$ , which of the following could be false?

(A)  $f'(c) = \frac{f(b) - f(a)}{b - a}$  for some  $c$  such that  $a < c < b$ .

(B)  $f'(c) = 0$  for some  $c$  such that  $a < c < b$ .

(C)  $f$  has a minimum value on  $a \leq x \leq b$ .

(D)  $f$  has a maximum value on  $a \leq x \leq b$ .

(E)  $\int_a^b f(x) dx$  exists.

5.  $\int_0^x \sin t \, dt =$

(A)  $\sin x$

(B)  $-\cos x$

(C)  $\cos x$

(D)  $\cos x - 1$

(E)  $1 - \cos x$

6. If  $x^2 + xy = 10$ , then when  $x = 2$ ,  $\frac{dy}{dx} =$

(A)  $-\frac{7}{2}$

(B)  $-2$

(C)  $\frac{2}{7}$

(D)  $\frac{3}{2}$

(E)  $\frac{7}{2}$

7.  $\int_1^e \left( \frac{x^2 - 1}{x} \right) dx =$

(A)  $e - \frac{1}{e}$

(B)  $e^2 - e$

(C)  $\frac{e^2}{2} - e + \frac{1}{2}$

(D)  $e^2 - 2$

(E)  $\frac{e^2}{2} - \frac{3}{2}$

8. Let  $f$  and  $g$  be differentiable functions with the following properties:

(i)  $g(x) > 0$  for all  $x$

(ii)  $f(0) = 1$

If  $h(x) = f(x)g(x)$  and  $h'(x) = f(x)g'(x)$ , then  $f(x) =$

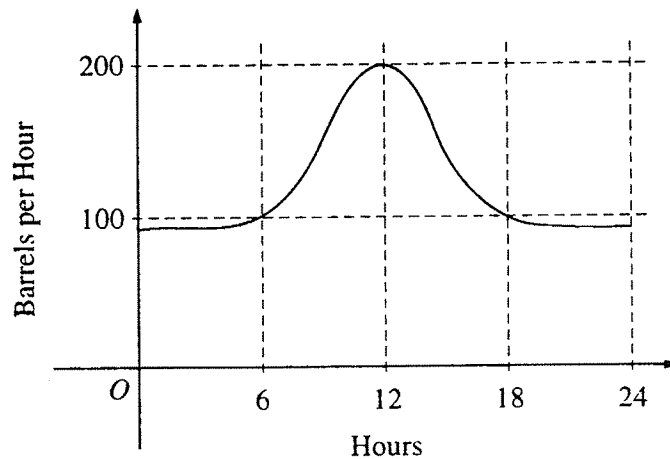
(A)  $f'(x)$

(B)  $g(x)$

(C)  $e^x$

(D)  $0$

(E)  $1$



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500      (B) 600      (C) 2,400      (D) 3,000      (E) 4,800

10. What is the instantaneous rate of change at  $x = 2$  of the function  $f$  given by  $f(x) = \frac{x^2 - 2}{x - 1}$ ?

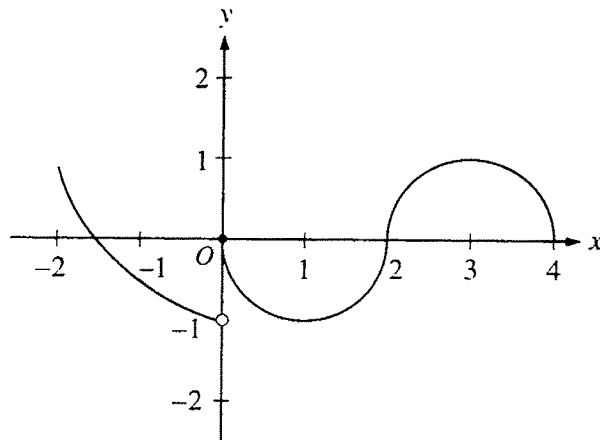
(A)  $-2$       (B)  $\frac{1}{6}$       (C)  $\frac{1}{2}$       (D)  $2$       (E)  $6$

11. If  $f$  is a linear function and  $0 < a < b$ , then  $\int_a^b f''(x) dx =$

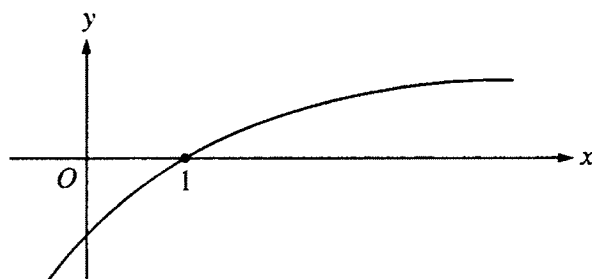
(A)  $0$       (B)  $1$       (C)  $\frac{ab}{2}$       (D)  $b - a$       (E)  $\frac{b^2 - a^2}{2}$

12. If  $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$  then  $\lim_{x \rightarrow 2} f(x)$  is

(A)  $\ln 2$       (B)  $\ln 8$       (C)  $\ln 16$       (D)  $4$       (E) nonexistent



13. The graph of the function  $f$  shown in the figure above has a vertical tangent at the point  $(2, 0)$  and horizontal tangents at the points  $(1, -1)$  and  $(3, 1)$ . For what values of  $x$ ,  $-2 < x < 4$ , is  $f$  not differentiable?
- (A) 0 only    (B) 0 and 2 only    (C) 1 and 3 only    (D) 0, 1, and 3 only    (E) 0, 1, 2, and 3
- 
14. A particle moves along the  $x$ -axis so that its position at time  $t$  is given by  $x(t) = t^2 - 6t + 5$ . For what value of  $t$  is the velocity of the particle zero?
- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5
- 
15. If  $F(x) = \int_0^x \sqrt{t^3 + 1} dt$ , then  $F'(2) =$
- (A) -3                      (B) -2                      (C) 2                      (D) 3                      (E) 18
- 
16. If  $f(x) = \sin(e^{-x})$ , then  $f'(x) =$
- (A)  $-\cos(e^{-x})$   
 (B)  $\cos(e^{-x}) + e^{-x}$   
 (C)  $\cos(e^{-x}) - e^{-x}$   
 (D)  $e^{-x} \cos(e^{-x})$   
 (E)  $-e^{-x} \cos(e^{-x})$



17. The graph of a twice-differentiable function  $f$  is shown in the figure above. Which of the following is true?

- (A)  $f(1) < f'(1) < f''(1)$
- (B)  $f(1) < f''(1) < f'(1)$
- (C)  $f'(1) < f(1) < f''(1)$
- (D)  $f''(1) < f(1) < f'(1)$
- (E)  $f''(1) < f'(1) < f(1)$

18. An equation of the line tangent to the graph of  $y = x + \cos x$  at the point  $(0, 1)$  is

- (A)  $y = 2x + 1$
- (B)  $y = x + 1$
- (C)  $y = x$
- (D)  $y = x - 1$
- (E)  $y = 0$

19. If  $f''(x) = x(x+1)(x-2)^2$ , then the graph of  $f$  has inflection points when  $x =$

- (A)  $-1$  only
- (B)  $2$  only
- (C)  $-1$  and  $0$  only
- (D)  $-1$  and  $2$  only
- (E)  $-1, 0,$  and  $2$  only

20. What are all values of  $k$  for which  $\int_{-3}^k x^2 dx = 0$ ?

- (A)  $-3$
- (B)  $0$
- (C)  $3$
- (D)  $-3$  and  $3$
- (E)  $-3, 0,$  and  $3$

21. If  $\frac{dy}{dt} = ky$  and  $k$  is a nonzero constant, then  $y$  could be

- (A)  $2e^{kty}$
- (B)  $2e^{kt}$
- (C)  $e^{kt} + 3$
- (D)  $kty + 5$
- (E)  $\frac{1}{2}ky^2 + \frac{1}{2}$

22. The function  $f$  is given by  $f(x) = x^4 + x^2 - 2$ . On which of the following intervals is  $f$  increasing?

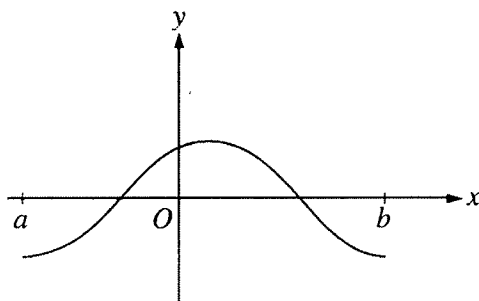
(A)  $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B)  $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

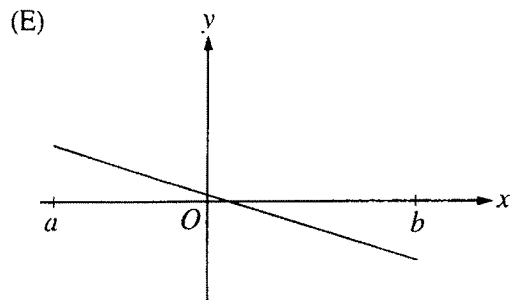
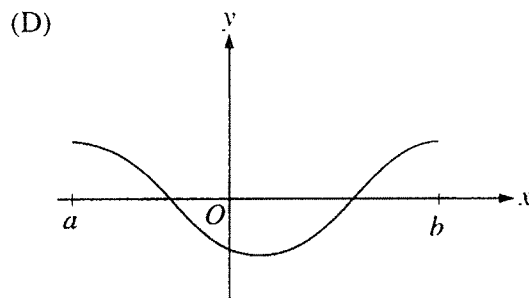
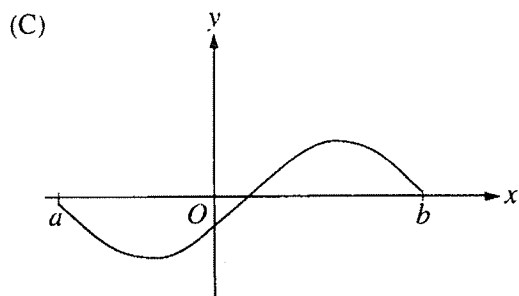
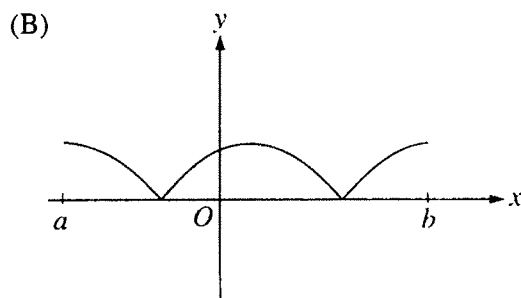
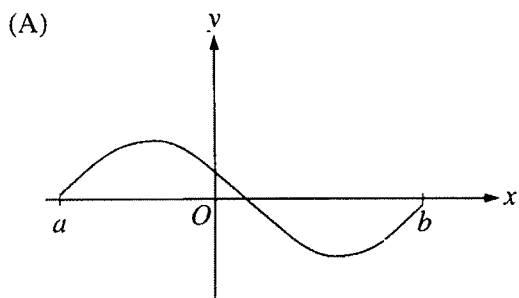
(C)  $(0, \infty)$

(D)  $(-\infty, 0)$

(E)  $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$



23. The graph of  $f$  is shown in the figure above. Which of the following could be the graph of the derivative of  $f$ ?



24. The maximum acceleration attained on the interval  $0 \leq t \leq 3$  by the particle whose velocity is given by  $v(t) = t^3 - 3t^2 + 12t + 4$  is

- (A) 9                      (B) 12                      (C) 14                      (D) 21                      (E) 40

25. What is the area of the region between the graphs of  $y = x^2$  and  $y = -x$  from  $x = 0$  to  $x = 2$ ?

- (A)  $\frac{2}{3}$                       (B)  $\frac{8}{3}$                       (C) 4                      (D)  $\frac{14}{3}$                       (E)  $\frac{16}{3}$

$x$	0	1	2
$f(x)$	1	$k$	2

26. The function  $f$  is continuous on the closed interval  $[0, 2]$  and has values that are given in the table above. The equation  $f(x) = \frac{1}{2}$  must have at least two solutions in the interval  $[0, 2]$  if  $k =$

- (A) 0                      (B)  $\frac{1}{2}$                       (C) 1                      (D) 2                      (E) 3

27. What is the average value of  $y = x^2\sqrt{x^3 + 1}$  on the interval  $[0, 2]$ ?

- (A)  $\frac{26}{9}$                       (B)  $\frac{52}{9}$                       (C)  $\frac{26}{3}$                       (D)  $\frac{52}{3}$                       (E) 24

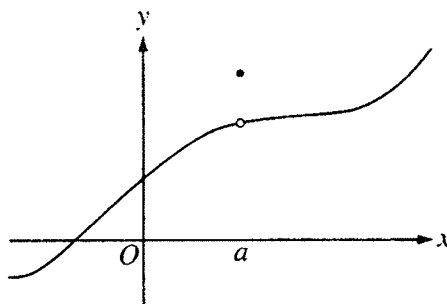
28. If  $f(x) = \tan(2x)$ , then  $f'\left(\frac{\pi}{6}\right) =$

- (A)  $\sqrt{3}$                       (B)  $2\sqrt{3}$                       (C) 4                      (D)  $4\sqrt{3}$                       (E) 8



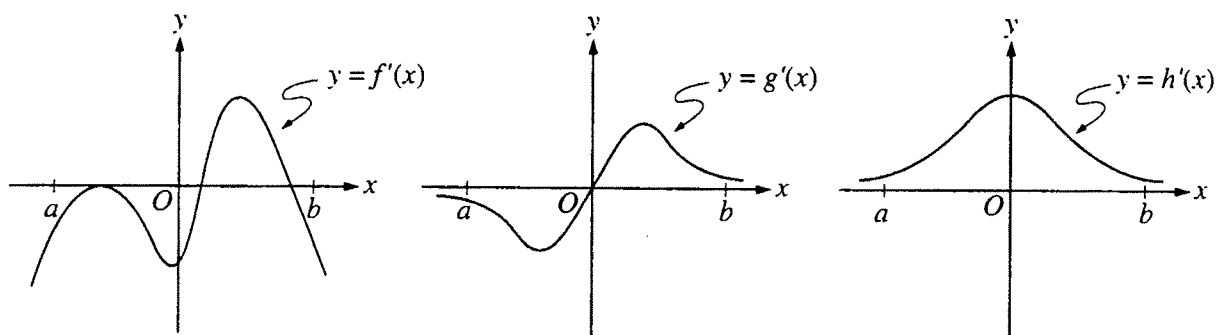
50 Minutes—Graphing Calculator Required

- Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.



76. The graph of a function  $f$  is shown above. Which of the following statements about  $f$  is false?
- (A)  $f$  is continuous at  $x = a$ .
- (B)  $f$  has a relative maximum at  $x = a$ .
- (C)  $x = a$  is in the domain of  $f$ .
- (D)  $\lim_{x \rightarrow a^+} f(x)$  is equal to  $\lim_{x \rightarrow a^-} f(x)$ .
- (E)  $\lim_{x \rightarrow a} f(x)$  exists.
- 
77. Let  $f$  be the function given by  $f(x) = 3e^{2x}$  and let  $g$  be the function given by  $g(x) = 6x^3$ . At what value of  $x$  do the graphs of  $f$  and  $g$  have parallel tangent lines?
- (A)  $-0.701$
- (B)  $-0.567$
- (C)  $-0.391$
- (D)  $-0.302$
- (E)  $-0.258$

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference  $C$ , what is the rate of change of the area of the circle, in square centimeters per second?
- (A)  $-(0.2)\pi C$   
 (B)  $-(0.1)C$   
 (C)  $-\frac{(0.1)C}{2\pi}$   
 (D)  $(0.1)^2 C$   
 (E)  $(0.1)^2 \pi C$



79. The graphs of the derivatives of the functions  $f$ ,  $g$ , and  $h$  are shown above. Which of the functions  $f$ ,  $g$ , or  $h$  have a relative maximum on the open interval  $a < x < b$ ?
- (A)  $f$  only  
 (B)  $g$  only  
 (C)  $h$  only  
 (D)  $f$  and  $g$  only  
 (E)  $f$ ,  $g$ , and  $h$

80. The first derivative of the function  $f$  is given by  $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$ . How many critical values does  $f$  have on the open interval  $(0,10)$ ?
- (A) One  
 (B) Three  
 (C) Four  
 (D) Five  
 (E) Seven

81. Let  $f$  be the function given by  $f(x) = |x|$ . Which of the following statements about  $f$  are true?

- I.  $f$  is continuous at  $x = 0$ .
- II.  $f$  is differentiable at  $x = 0$ .
- III.  $f$  has an absolute minimum at  $x = 0$ .

(A) I only    (B) II only    (C) III only    (D) I and III only    (E) II and III only

82. If  $f$  is a continuous function and if  $F'(x) = f(x)$  for all real numbers  $x$ , then  $\int_1^3 f(2x) dx =$

- (A)  $2F(3) - 2F(1)$
- (B)  $\frac{1}{2}F(3) - \frac{1}{2}F(1)$
- (C)  $2F(6) - 2F(2)$
- (D)  $F(6) - F(2)$
- (E)  $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

83. If  $a \neq 0$ , then  $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$  is

- (A)  $\frac{1}{a^2}$
- (B)  $\frac{1}{2a^2}$
- (C)  $\frac{1}{6a^2}$
- (D) 0
- (E) nonexistent

84. Population  $y$  grows according to the equation  $\frac{dy}{dt} = ky$ , where  $k$  is a constant and  $t$  is measured in years. If the population doubles every 10 years, then the value of  $k$  is

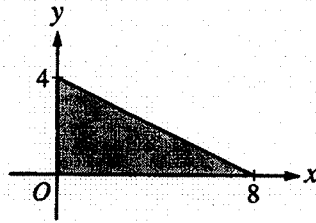
- (A) 0.069
- (B) 0.200
- (C) 0.301
- (D) 3.322
- (E) 5.000

$x$	2	5	7	8
$f(x)$	10	30	40	20

85. The function  $f$  is continuous on the closed interval  $[2, 8]$  and has values that are given in the table above. Using the subintervals  $[2, 5]$ ,  $[5, 7]$ , and  $[7, 8]$ , what is the trapezoidal approximation of

$$\int_2^8 f(x) dx?$$

- (A) 110                      (B) 130                      (C) 160                      (D) 190                      (E) 210



86. The base of a solid is a region in the first quadrant bounded by the  $x$ -axis, the  $y$ -axis, and the line  $x + 2y = 8$ , as shown in the figure above. If cross sections of the solid perpendicular to the  $x$ -axis are semicircles, what is the volume of the solid?

- (A) 12.566                      (B) 14.661                      (C) 16.755                      (D) 67.021                      (E) 134.041

87. Which of the following is an equation of the line tangent to the graph of  $f(x) = x^4 + 2x^2$  at the point where  $f'(x) = 1$ ?

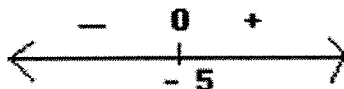
- (A)  $y = 8x - 5$   
 (B)  $y = x + 7$   
 (C)  $y = x + 0.763$   
 (D)  $y = x - 0.122$   
 (E)  $y = x - 2.146$

88. Let  $F(x)$  be an antiderivative of  $\frac{(\ln x)^3}{x}$ . If  $F(1) = 0$ , then  $F(9) =$

- (A) 0.048                      (B) 0.144                      (C) 5.827                      (D) 23.308                      (E) 1,640.250

89. If  $g$  is a differentiable function such that  $g(x) < 0$  for all real numbers  $x$  and if  $f'(x) = (x^2 - 4)g(x)$ , which of the following is true?
- (A)  $f$  has a relative maximum at  $x = -2$  and a relative minimum at  $x = 2$ .  
(B)  $f$  has a relative minimum at  $x = -2$  and a relative maximum at  $x = 2$ .  
(C)  $f$  has relative minima at  $x = -2$  and at  $x = 2$ .  
(D)  $f$  has relative maxima at  $x = -2$  and at  $x = 2$ .  
(E) It cannot be determined if  $f$  has any relative extrema.
- 
90. If the base  $b$  of a triangle is increasing at a rate of 3 inches per minute while its height  $h$  is decreasing at a rate of 3 inches per minute, which of the following must be true about the area  $A$  of the triangle?
- (A)  $A$  is always increasing.  
(B)  $A$  is always decreasing.  
(C)  $A$  is decreasing only when  $b < h$ .  
(D)  $A$  is decreasing only when  $b > h$ .  
(E)  $A$  remains constant.
- 
91. Let  $f$  be a function that is differentiable on the open interval  $(1, 10)$ . If  $f(2) = -5$ ,  $f(5) = 5$ , and  $f(9) = -5$ , which of the following must be true?
- I.  $f$  has at least 2 zeros.  
II. The graph of  $f$  has at least one horizontal tangent.  
III. For some  $c$ ,  $2 < c < 5$ ,  $f(c) = 3$ .
- (A) None  
(B) I only  
(C) I and II only  
(D) I and III only  
(E) I, II, and III
- 
92. If  $0 \leq k < \frac{\pi}{2}$  and the area under the curve  $y = \cos x$  from  $x = k$  to  $x = \frac{\pi}{2}$  is 0.1, then  $k =$
- (A) 1.471      (B) 1.414      (C) 1.277      (D) 1.120      (E) 0.436

1. D  $y' = x^2 + 10x \rightarrow y'' = 2x + 10$  Inflection points happen when second

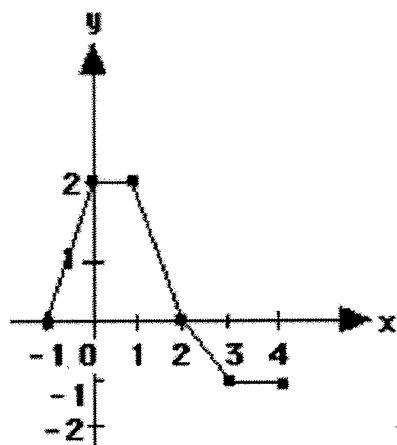


derivative is 0.  $2x + 10 = 0 \rightarrow x = \boxed{-5}$

Second

derivative is negative for

$x < -5$  and positive for  $x > -5$  so it is an inflection point.



2. B Use areas to get integral. From  $-1$  to  $2$  area is a

trapezoid, area  $\frac{1}{2}(2)(3+1) = 4$  and from  $2$  to  $4$  is a trapezoid

$\frac{1}{2}(-1)(2+1) = -1.5$  so answer is  $\boxed{2.5}$

3. C  $\int_1^2 (x^{-2}) dx = \left( \frac{x^{-1}}{-1} \right) \Big|_1^2 = \left( \frac{-1}{x} \right) \Big|_1^2 = -\frac{1}{2} - (-1) = \boxed{\frac{1}{2}}$

4. B A gives the conditions for the Mean Value Theorem. B This is Rollés Theorem which has the added condition that  $f(a) = f(b) = 0$ , so this is false. The conditions given make C and D true—absolute maximum and minimum. E The conditions are what are necessary for the integral to exist.

5. E  $-\cos t \Big|_0^x = -\cos x - (-\cos 0) = \boxed{1 - \cos x}$

6. A  $2^2 + 2y = 10 \rightarrow y = 3$  Take derivative implicitly  $2x + x \frac{dy}{dx} + y = 0$ . Solve for

$\frac{dy}{dx} \quad x \frac{dy}{dx} = -(2x + y) \rightarrow \frac{dy}{dx} = \frac{-(2x + y)}{x}$  At  $(2, 3)$ ,  $\frac{dy}{dx} = \frac{-(2(2) + 3)}{2} = \boxed{-\frac{7}{2}}$

7. E Long divide giving  $\int_1^e \left( x - \frac{1}{x} \right) dx = \int_1^e (x - x^{-1}) dx = \left[ \frac{x^2}{2} - \ln x \right]_1^e =$

$$\frac{e^2}{2} - \ln e - \left( \frac{1}{2} - \ln 1 \right) = \frac{e^2}{2} - 1 - \left( \frac{1}{2} - 0 \right) = \boxed{\frac{e^2}{2} - \frac{3}{2}}$$

8. E Using the product rule  $h'(x) = f(x)g'(x) + f'(x)g(x)$ , but  $h'(x) = f(x)g'(x)$  as given. Thus,  $f'(x)g(x) = 0$ . Since  $g(x) > 0$ ,  $f'(x) = 0 \rightarrow f(x)$  is a constant.

Thus, since  $f(0) = 1 \rightarrow f(1) = \boxed{1}$ . (Only constants have 0 derivative).

9. D Look at the graph and the area under it is about 5 of the boxes that are 6 hours by 100 barrels/hour =  $\boxed{3000}$  barrels

10. D Instantaneous rate of change is the derivative at the point.

$$f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2} = \frac{x^2 - 2x + 2}{(x-1)^2} \rightarrow f'(2) = \frac{2^2 - 2(2) + 2}{(2-1)^2} = \boxed{2}$$

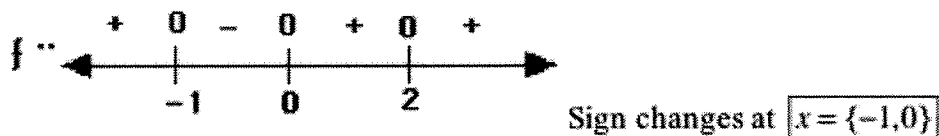
11. A If  $f(x)$  is linear, the  $f(x) = mx + b \rightarrow f'(x) = m \rightarrow f''(x) = 0$ . The integral of 0 is  $\boxed{0}$ .

12. E Left limit  $\lim_{x \rightarrow 2^-} (\ln x) = \ln 2$  Right limit  $\lim_{x \rightarrow 2^+} (x^2 \ln x) = 4 \ln 2$   $\boxed{\ln 2 \neq 4 \ln 2}$

16. E Using the chain rule  $\cos(e^{-x}) \frac{d}{dx}(e^{-x}) = \boxed{-(e^{-x})\cos(e^{-x})}$
17. D The graph shows  $f(1) = 0$ . Since function is increasing at  $x = 1, f'(x) > 0$ . Since function is concave down at  $x = 1, f''(x) < 0$ . In terms of the answers, this means  $\boxed{f''(1) < f(1) < f'(1)}$ .

18. B Derivative is  $y' = 1 - \sin x$ . So slope,  $m$  at  $(0, 1)$  would be  $1 - 0 = 1$ .  
 $y = mx + b \rightarrow y = 1x + 1 \rightarrow \boxed{y = x + 1}$ . Remember  $(0, b) \rightarrow b$  is y-intercept.

19. C Inflection points have 2<sup>nd</sup> derivative = 0 and change of signs.  
 $f''(x) = 0$  at  $x = \{-1, 0, 2\}$  Make a 2<sup>nd</sup> derivative chart as below.



20. A  $\frac{x^3}{3} \Big|_{-3}^k = 0 \rightarrow \frac{k^3}{3} - \frac{(-3)^3}{3} = 0 \rightarrow k^3 + 27 = 0 \rightarrow k^3 = -27 \rightarrow \boxed{k = -3}$

21. B  $\frac{dy}{y} = k dx \rightarrow \int \frac{dy}{y} = \int k dx \rightarrow \ln y = kx + C \rightarrow e^{\ln y} = e^{kx+C} \rightarrow$   
 $y = e^{kx+C} \rightarrow y = e^{kx} \cdot e^C$  However,  $e^C$  is just a constant, call it  
 $D \rightarrow y = De^{kx}$ . Let  $D = 2 \rightarrow \boxed{y = 2e^{kx}}$

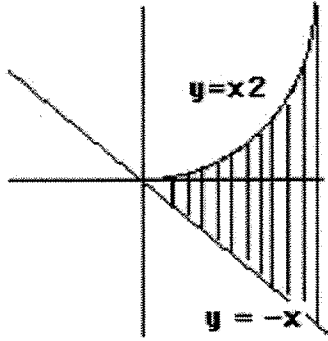
22. C  $f'(x) = 4x^3 + 2x = 2x(x^2 + 1)$  Set derivative to 0.  $f'(x) = 0$  at  $x = 0$ . A function increases when 1<sup>st</sup> derivative is positive. Make 1<sup>st</sup> derivative chart.



23. A When a function increases, its 1<sup>st</sup> derivative is positive (above the x-axis). When a function decreases, its 1<sup>st</sup> derivative is negative (below the x-axis). The derivative would be 0 at the maximum of the function. So the 1<sup>st</sup> derivative would be above the x-axis from  $a$  to the maximum point of the function, on the x-axis at the function's maximum point, then below the x-axis for the rest of the time to  $b$ .  $\boxed{\text{Only graph A exhibits this.}}$



24. D  $v'(t) = a(t) = 3t^2 - 6t + 12$ . To find the absolute maximum of the acceleration, evaluate the value of the acceleration at the endpoints ( $t = 0$  and  $t = 3$ ) and any critical points on the interval  $[0,3]$ . The largest value for  $a(t)$  of these will be the absolute maximum of  $a(t)$  on the interval.  
For the critical points of  $a(t)$  take its derivative and set it equal to 0.  
 $a'(t) = 6t - 6 \rightarrow 6t - 6 = 0 \rightarrow t = 1$ .  
Evaluate  $a(t)$  at  $t = 0, 1, 3$ .  $(0,12), (1,9), (3,21)$ . The largest value is 21. **D**



25. D

$$A = \int_0^2 (x^2 - (-x)) dx = \int_0^2 (x^2 + x) dx = \left. \frac{x^3}{3} + \frac{x^2}{2} \right|_0^2 = \left( \frac{8}{3} + \frac{4}{2} - (0+0) \right) = \frac{14}{3}$$

26. A By the Intermediate Value Theorem, if  $k$  were some value less than  $\frac{1}{2}$ , then the function would have to equal  $\frac{1}{2}$  between  $0 < x < 1$  and again between  $1 < x < 2$ . The only value less than  $\frac{1}{2}$  is **0**.

27. A The average value on  $[0,2]$  of

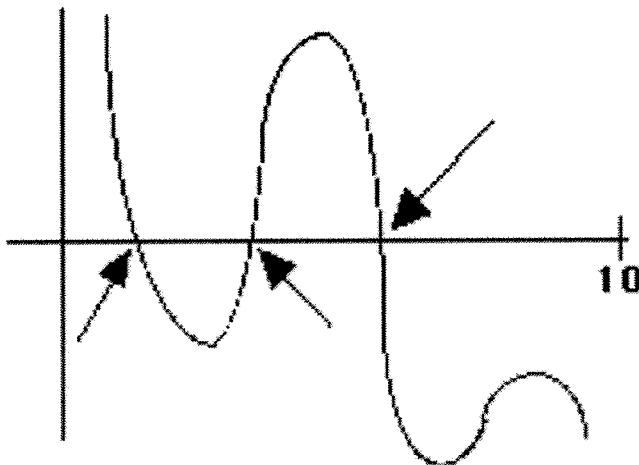
$$y = \frac{1}{2-0} \int_0^2 (x^2 \sqrt{x^3+1}) dx \quad \text{Let } u = x^3+1 \quad du = 3x^2 \rightarrow \frac{1}{2-0} \int_0^2 (x^2 \sqrt{x^3+1}) dx = \frac{1}{2} \left( \frac{2}{9} (x^3+1)^{3/2} \right) \Big|_0^2 = \frac{1}{9} \left[ (9^{3/2} - 1^{3/2}) - (0) \right] = \frac{1}{9} (27 - 1) = \frac{26}{9}$$

28. E  $f'(x) = \sec^2(2x) (2) \rightarrow f'\left(\frac{\pi}{6}\right) = 2 \sec^2\left(2\left(\frac{\pi}{6}\right)\right) = 2 \sec^2\left(\frac{\pi}{3}\right) = 2(2)^2 = \mathbf{8}$

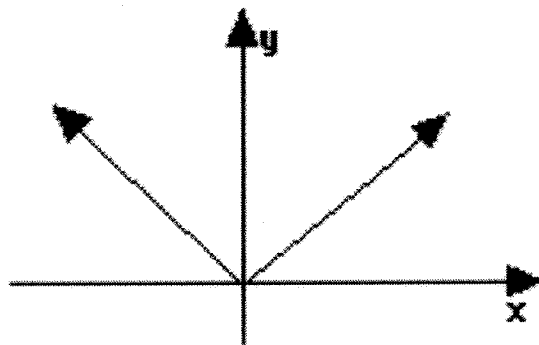
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76. A B is true as in every neighborhood of  $x = a$   $f(a)$  is greater than any other  $f(x)$ . C is true because  $f(a)$  exists. D is true because both sides of graph converge on the same point at  $x = a$ . E is true because, again, both sides of graph converge on the same point at  $x = a$ . **A** is false because  $\lim_{x \rightarrow a} f(x) \neq f(a)$ .
77. C For the tangent lines to be parallel,  
 $f'(x) = g'(x)$   $f'(x) = 6e^{2x}$   $g'(x) = 18x^2 \rightarrow 6e^{2x} = 18x^2 \rightarrow 6e^{2x} - 18x^2 = 0$   
 Solving on calculator yields  **$x = -.391$**
78. B  $A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$  But,  $C = 2\pi r \rightarrow \frac{dA}{dt} = C \frac{dr}{dt} = \mathbf{-.1C}$
79. A For a function to have a relative maximum, the 1<sup>st</sup> derivative must equal 0 and the 2<sup>nd</sup> derivative must be negative, meaning the 1<sup>st</sup> derivative must be decreasing. On  $a < x < b$ ,  $g'(x)$  is increasing when it is 0,  $h'(x)$  is not zero. That means that **only  $f(x)$**  has the required relative maximum.

80. B Critical points occur where the 1<sup>st</sup> derivative is 0. Graphing the function on the given interval



yields 3 such points.



81. D

II At  $x = 0$ , the  $f(0)$  is less than all other  $f(x)$ , so it is an absolute maximum. III  $\lim_{x \rightarrow 0^-} f'(x) = \lim_{x \rightarrow 0^+} f'(x)$  so  $f$  is not differentiable. I The  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$  so  $f(x)$  is continuous at  $x = 0$ .

82. E 1<sup>st</sup> Fundamental Theorem of Calculus.

$$\frac{d}{dx}(F(2x)) = F'(2x)(2) = 2f'(2x) \rightarrow \int f'(2x)dx = \frac{1}{2}F(2x) + C \rightarrow$$

$$\int_1^3 (f(2x))dx = \frac{1}{2}[F(2(3)) - F(2(1))] = \boxed{\frac{1}{2}F(6) - \frac{1}{2}F(2)}$$

$$83. \text{ B } \lim_{x \rightarrow a} \frac{(x^2 - a^2)}{(x^2 - a^2)(x^2 + a^2)} = \lim_{x \rightarrow a} \frac{1}{(x^2 + a^2)} = \boxed{\frac{1}{2a^2}}$$

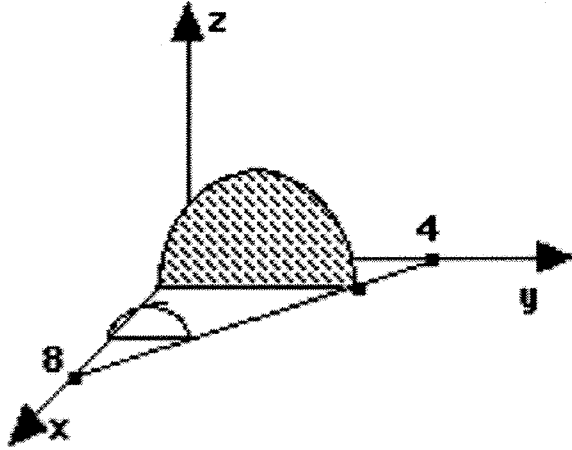
84. A  $\frac{dy}{y} = k dx \rightarrow \int \frac{dy}{y} = \int k dx \rightarrow \ln y = kx + C$  Use point  $(0, y_0)$  to find C, then point  $(10, 2y_0)$  to find k.  $\ln(y_0) = k(0) + C \rightarrow C = \ln(y_0)$

$$\ln(2y_0) = 10k + \ln(y_0) \rightarrow \ln(2y_0) - \ln(y_0) = 10k \rightarrow \ln\left(\frac{2y_0}{y_0}\right) = 10k \rightarrow k = \frac{\ln 2}{10} =$$

$$\boxed{0.069}$$

$$85. \text{ C } \text{Area} = \frac{1}{2}(3)(10 + 30) + \frac{1}{2}(2)(30 + 40) + \frac{1}{2}(1)(40 + 20) = 60 + 70 + 30 = \boxed{160}.$$

86. C



$$x + 2y = 8 \rightarrow y = 4 - \frac{1}{2}x \rightarrow r = 2 - \frac{1}{4}x \quad V = \frac{1}{2}\pi \int_0^8 \left(2 - \frac{1}{4}x\right)^2 dx = \boxed{16.755}$$

87. D  $f'(x) = 4x^3 + 4x \rightarrow f'(x) = 1 = 4x^3 + 4x \rightarrow 4x^3 + 4x - 1 = 0$  Using calculator to solve,  $x = .237$   $f(.237) = .115 \rightarrow y - .115 = 1(x - .237) \rightarrow \boxed{y = x - .122}$

88. C  $u = \ln x \quad du = \frac{dx}{x} \rightarrow F(x) = \left( \frac{(\ln x)^4}{4} \right) + C \rightarrow$

$$F(1) = \left( \frac{(\ln 1)^4}{4} \right) + C \rightarrow 0 = 0 + C \rightarrow C = 0 \rightarrow F(9) = \left( \frac{(\ln 9)^4}{4} \right) = \boxed{5.827}$$

89. B  $f'(x) = 0 = (x^2 - 4)g(x)$  with  $g(x) < 0 \rightarrow x = \{-2, 2\}$

$$f''(x) = (x^2 - 4)g'(x) + 2x g(x) \rightarrow f''(-2) = 0 - 4g(x) \text{ and } f''(2) = 0 + 4g(x)$$

Since  $g(x) < 0 \rightarrow f''(-2) > 0$  and  $f''(2) < 0 \rightarrow f'(-2) = 0, f''(-2) > 0 \rightarrow$

**relative minimum at  $x = -2$**  and

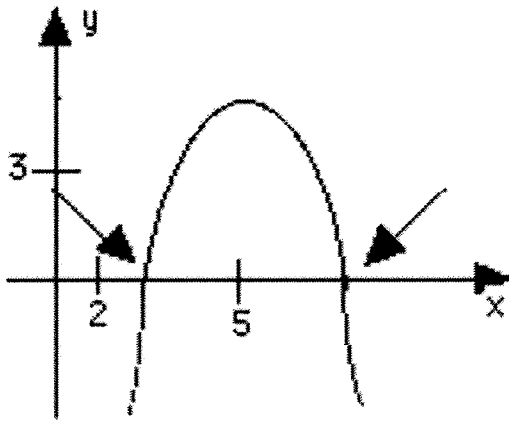
**$f'(2) = 0, f''(2) < 0 \rightarrow$  relative maximum at  $x=2$**

90. D  $A_s = \frac{1}{2}bh \rightarrow \frac{dA_s}{dt} = \frac{1}{2} \left( b \frac{dh}{dt} + h \frac{db}{dt} \right) = \frac{1}{2}(-3b + 3h)$  If

$b > h \rightarrow \frac{dA_s}{dt} < 0$   $b = h \rightarrow \frac{dA_s}{dt} = 0$  1<sup>st</sup> derivative is negative means function

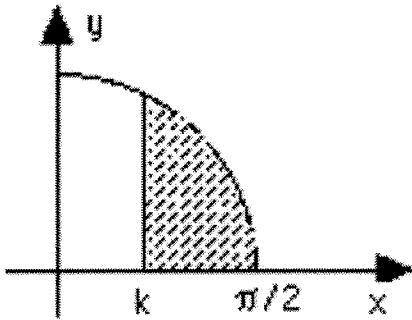
is decreasing. 1<sup>st</sup> derivative is positive means the function is increasing. This

means that **the function is decreasing when  $b > h$**



91. E Graph it. One can see that it has **I** a horizontal tangent, it **II** has 2 zeroes, and by the intermediate value theorem, there **III** there must be a  $c$ ,  $2 < c < 5$  such that,  $c = 3$ , since  $-5 < f(x) < 5$  on the interval  $2 < c < 5$ .

92. D



$$\int_k^{\pi/2} (\cos x) dx = .1 \rightarrow \sin\left(\frac{\pi}{2}\right) - \sin k = .1 \rightarrow \sin k = \sin\left(\frac{\pi}{2}\right) - .1 \rightarrow \sin k = .9 \rightarrow \boxed{k = 1.120}$$

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