

1997 Calculus AB Solutions: Part A

1. C $\int_1^2 (4x^3 - 6x) dx = (x^4 - 3x^2) \Big|_1^2 = (16 - 12) - (1 - 3) = 6$
2. A $f(x) = x(2x - 3)^{\frac{1}{2}}$; $f'(x) = (2x - 3)^{\frac{1}{2}} + x(2x - 3)^{-\frac{1}{2}} = (2x - 3)^{-\frac{1}{2}}(3x - 3) = \frac{(3x - 3)}{\sqrt{2x - 3}}$
3. C $\int_a^b (f(x) + 5) dx = \int_a^b f(x) dx + 5 \int_a^b 1 dx = a + 2b + 5(b - a) = 7b - 4a$
4. D $f(x) = -x^3 + x + \frac{1}{x}$; $f'(x) = -3x^2 + 1 - \frac{1}{x^2}$; $f'(-1) = -3(-1)^2 + 1 - \frac{1}{(-1)^2} = -3 + 1 - 1 = -3$
5. E $y = 3x^4 - 16x^3 + 24x^2 + 48$; $y' = 12x^3 - 48x^2 + 48x$; $y'' = 36x^2 - 96x + 48 = 12(3x - 2)(x - 2)$
 $y'' < 0$ for $\frac{2}{3} < x < 2$, therefore the graph is concave down for $\frac{2}{3} < x < 2$
6. C $\frac{1}{2} \int e^{\frac{t}{2}} dt = e^{\frac{t}{2}} + C$
7. D $\frac{d}{dx} \cos^2(x^3) = 2 \cos(x^3) \left(\frac{d}{dx} (\cos(x^3)) \right) = 2 \cos(x^3) (-\sin(x^3)) \left(\frac{d}{dx} (x^3) \right)$
 $= 2 \cos(x^3) (-\sin(x^3)) (3x^2)$
8. C The bug change direction when v changes sign. This happens at $t = 6$.
9. B Let A_1 be the area between the graph and t -axis for $0 \leq t \leq 6$, and let A_2 be the area between the graph and the t -axis for $6 \leq t \leq 8$. Then $A_1 = 12$ and $A_2 = 1$. The total distance is $A_1 + A_2 = 13$.
10. E $y = \cos(2x)$; $y' = -2 \sin(2x)$; $y' \left(\frac{\pi}{4} \right) = -2$ and $y \left(\frac{\pi}{4} \right) = 0$; $y = -2 \left(x - \frac{\pi}{4} \right)$
11. E Since f' is positive for $-2 < x < 2$ and negative for $x < -2$ and for $x > 2$, we are looking for a graph that is increasing for $-2 < x < 2$ and decreasing otherwise. Only option E.
12. B $y = \frac{1}{2}x^2$; $y' = x$; We want $y' = \frac{1}{2} \Rightarrow x = \frac{1}{2}$. So the point is $\left(\frac{1}{2}, \frac{1}{8} \right)$.

13. A $f'(x) = \frac{|4-x^2|}{x-2}$; f is decreasing when $f' < 0$. Since the numerator is non-negative, this is only when the denominator is negative. Only when $x < 2$.
14. C $f(x) \approx L(x) = 2 + 5(x-3)$; $L(x) = 0$ if $0 = 5x - 13 \Rightarrow x = 2.6$
15. B Statement B is true because $\lim_{x \rightarrow a^-} f(x) = 2 = \lim_{x \rightarrow a^+} f(x)$. Also, $\lim_{x \rightarrow b} f(x)$ does not exist because the left- and right-sided limits are not equal, so neither (A), (C), nor (D) are true.
16. D The area of the region is given by $\int_{-2}^2 (5 - (x^2 + 1)) dx = 2(4x - \frac{1}{3}x^3) \Big|_0^2 = 2(8 - \frac{8}{3}) = \frac{32}{3}$
 $5 - x^2 - 1$
 $-x^2 + 4$
17. A $x^2 + y^2 = 25$; $2x + 2y \cdot y' = 0$; $x + y \cdot y' = 0$; $y'(4, 3) = -\frac{4}{3}$;
 $x + y \cdot y' = 0 \Rightarrow 1 + y \cdot y'' + y' \cdot y' = 0$; $1 + (3)y'' + (-\frac{4}{3}) \cdot (-\frac{4}{3}) = 0$; $y'' = -\frac{25}{27}$
18. C $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx$ is of the form $\int e^u du$ where $u = \tan x$.
 $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx = e^{\tan x} \Big|_0^{\frac{\pi}{4}} = e^1 - e^0 = e - 1$
19. D $f(x) = \ln|x^2 - 1|$; $f'(x) = \frac{1}{x^2 - 1} \cdot \frac{d}{dx}(x^2 - 1) = \frac{2x}{x^2 - 1}$
20. E $\frac{1}{8} \int_{-3}^5 \cos x dx = \frac{1}{8}(\sin 5 - \sin(-3)) = \frac{1}{8}(\sin 5 + \sin 3)$; Note: Since the sine is an odd function, $\sin(-3) = -\sin(3)$.
21. E $\lim_{x \rightarrow 1} \frac{x}{\ln x}$ is nonexistent since $\lim_{x \rightarrow 1} \ln x = 0$ and $\lim_{x \rightarrow 1} x \neq 0$.
22. D $f(x) = (x^2 - 3)e^{-x}$; $f'(x) = e^{-x}(-x^2 + 2x + 3) = -e^{-x}(x-3)(x+1)$; $f'(x) > 0$ for $-1 < x < 3$
23. A Disks where $r = x$. $V = \pi \int_0^2 x^2 dy = \pi \int_0^2 y^4 dy = \frac{\pi}{5} y^5 \Big|_0^2 = \frac{32\pi}{5}$

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24. B Let $[0,1]$ be divided into 50 subintervals. $\Delta x = \frac{1}{50}$; $x_1 = \frac{1}{50}, x_2 = \frac{2}{50}, x_3 = \frac{3}{50}, \dots, x_{50} = 1$

Using $f(x) = \sqrt{x}$, the right Riemann sum $\sum_{i=1}^{50} f(x_i)\Delta x$ is an approximation for $\int_0^1 \sqrt{x} dx$.

25. A Use the technique of antiderivatives by parts, which was removed from the AB Course Description in 1998.

$$u = x \quad dv = \sin 2x dx$$

$$du = dx \quad v = -\frac{1}{2} \cos 2x$$

$$\int x \sin(2x) dx = -\frac{1}{2} x \cos(2x) + \int \frac{1}{2} \cos(2x) dx = -\frac{1}{2} x \cos(2x) + \frac{1}{4} \sin(2x) + C$$

76. E $f(x) = \frac{e^{2x}}{2x}$; $f'(x) = \frac{2e^{2x} \cdot 2x - 2e^{2x}}{4x^2} = \frac{e^{2x}(2x-1)}{2x^2}$

77. D $y = x^3 + 6x^2 + 7x - 2\cos x$. Look at the graph of $y'' = 6x + 12 + 2\cos x$ in the window $[-3, -1]$ since that domain contains all the option values. y'' changes sign at $x = -1.89$.

78. D $F(3) - F(0) = \int_0^3 f(x) dx = \int_0^1 f(x) dx + \int_1^3 f(x) dx = 2 + 2.3 = 4.3$

(Count squares for $\int_0^1 f(x) dx$)

79. C The stem of the questions means $f'(2) = 5$. Thus f is differentiable at $x = 2$ and therefore continuous at $x = 2$. We know nothing of the continuity of f' . I and II only.

80. A $f(x) = 2e^{4x^2}$; $f'(x) = 16xe^{4x^2}$; We want $16xe^{4x^2} = 3$. Graph the derivative function and the function $y = 3$, then find the intersection to get $x = 0.168$.

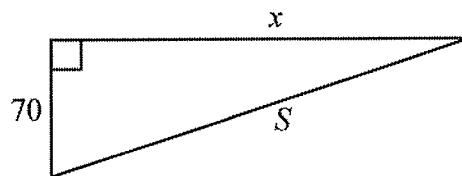
81. A Let x be the distance of the train from the

crossing. Then $\frac{dx}{dt} = 60$.

$$S^2 = x^2 + 70^2 \Rightarrow 2S \frac{dS}{dt} = 2x \frac{dx}{dt} \Rightarrow \frac{dS}{dt} = \frac{x}{S} \frac{dx}{dt}$$

After 4 seconds, $x = 240$ and so $S = 250$.

Therefore $\frac{dS}{dt} = \frac{240}{250}(60) = 57.6$



82. B $P(x) = 2x^2 - 8x$; $P'(x) = 4x - 8$; P' changes from negative to positive at $x = 2$. $P(2) = -8$

83. C $\cos x = x$ at $x = 0.739085$. Store this in A . $\int_0^A (\cos x - x) dx = 0.400$

84. C Cross sections are squares with sides of length y .

$$\text{Volume} = \int_1^e y^2 dx = \int_1^e \ln x dx = (x \ln x - x) \Big|_1^e = (e \ln e - e) - (0 - 1) = 1$$

85. C Look at the graph of f' and locate where the y changes from positive to negative. $x = 0.91$

86. A $f(x) = \sqrt{x}$; $f'(x) = \frac{1}{2\sqrt{x}}$; $\frac{1}{2\sqrt{c}} = 2 \cdot \frac{1}{2\sqrt{1}} \Rightarrow c = \frac{1}{4}$

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87. B $a(t) = t + \sin t$ and $v(0) = -2 \Rightarrow v(t) = \frac{1}{2}t^2 - \cos t - 1$; $v(t) = 0$ at $t = 1.48$

88. E $f(x) = \int_a^x h(x) dx \Rightarrow f(a) = 0$, therefore only (A) or (E) are possible. But $f'(x) = h(x)$ and therefore f is differentiable at $x = b$. This is true for the graph in option (E) but not in option (A) where there appears to be a corner in the graph at $x = b$. Also, Since h is increasing at first, the graph of f must start out concave up. This is also true in (E) but not (A).

89. B $T = \frac{1}{2} \cdot \frac{1}{2} (3 + 2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8 + 13) = 12$

90.	D	$F(x) = \frac{1}{2} \sin^2 x$	$F'(x) = \sin x \cos x$	Yes
		$F(x) = \frac{1}{2} \cos^2 x$	$F'(x) = -\cos x \sin x$	No
		$F(x) = -\frac{1}{4} \cos(2x)$	$F'(x) = \frac{1}{2} \sin(2x) = \sin x \cos x$	Yes